Rigidity of 2D and 3D quasicrystal frameworks

23.04.2020

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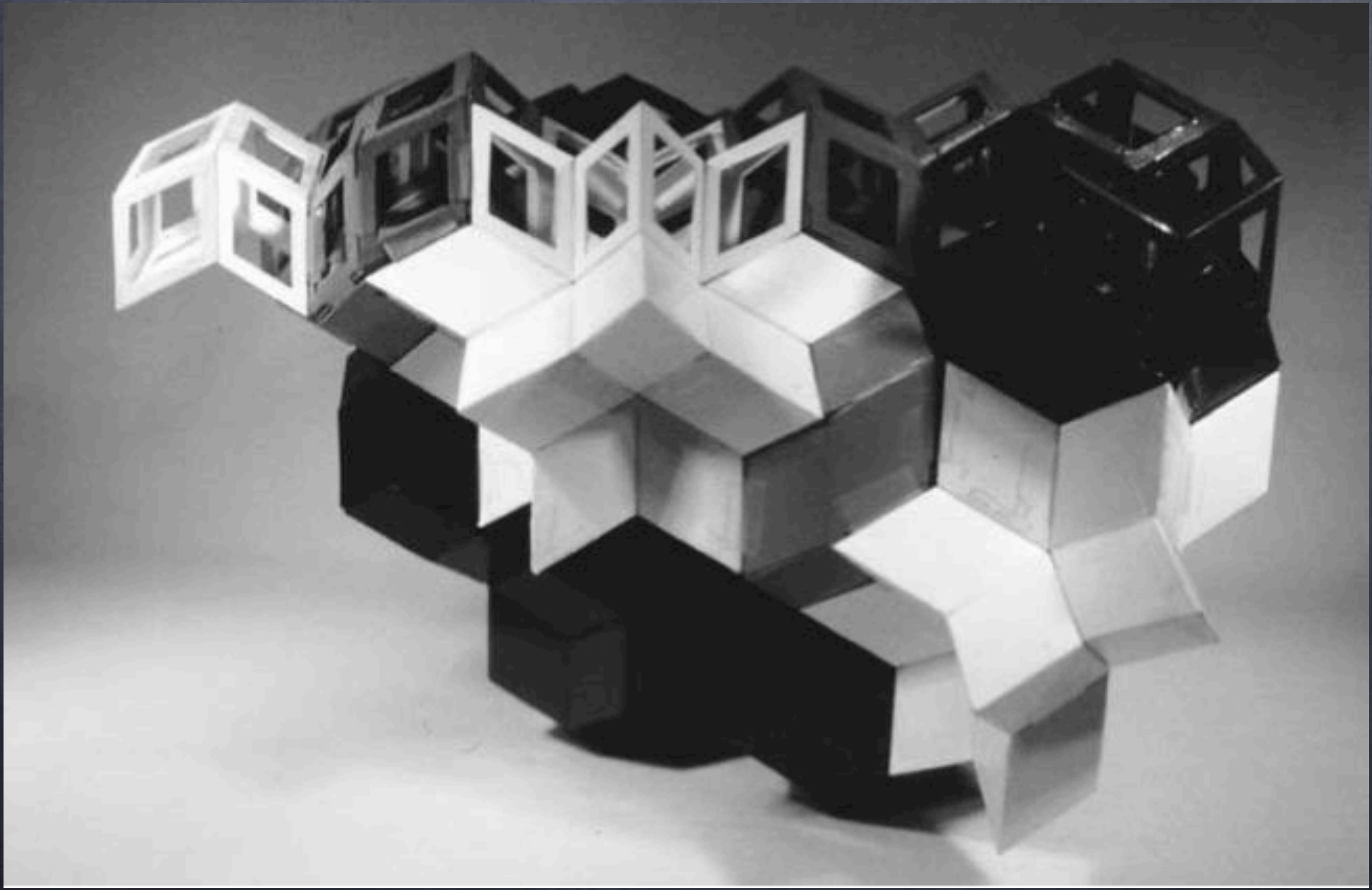
Collaborators - Illinois Geometry Lab

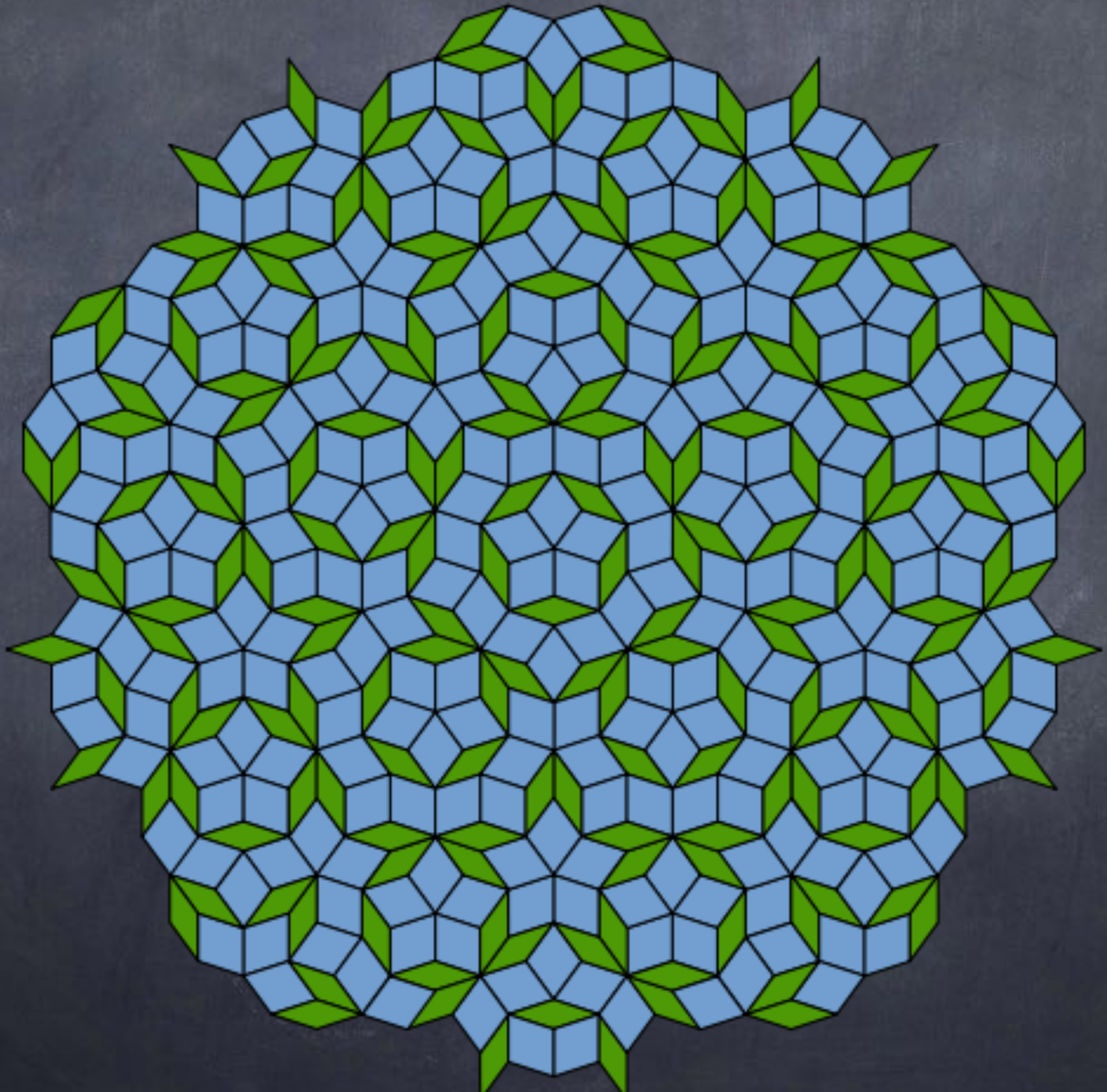
Tony Robbin

COAST, Sculpture, Danish Technical University

Quasicrystal composition



Penrose frameworks



Def: A **bar-joint framework** is a pair (G,p) where $p: V(G) \to \mathbb{R}^d$ is a map such that $p(u) \neq p(v)$ for all $(u,v) \in E(G)$.

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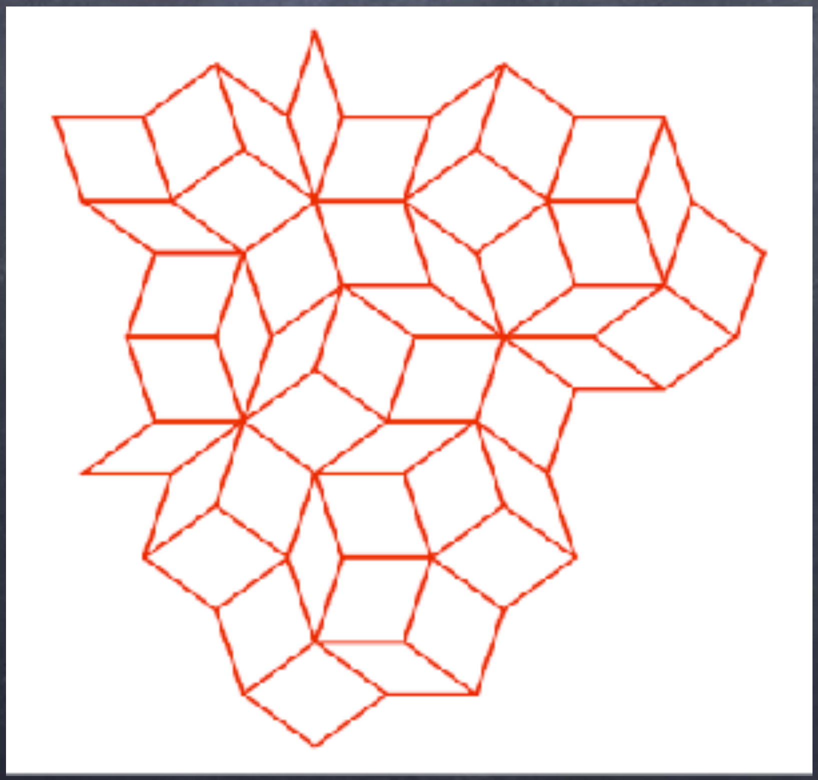
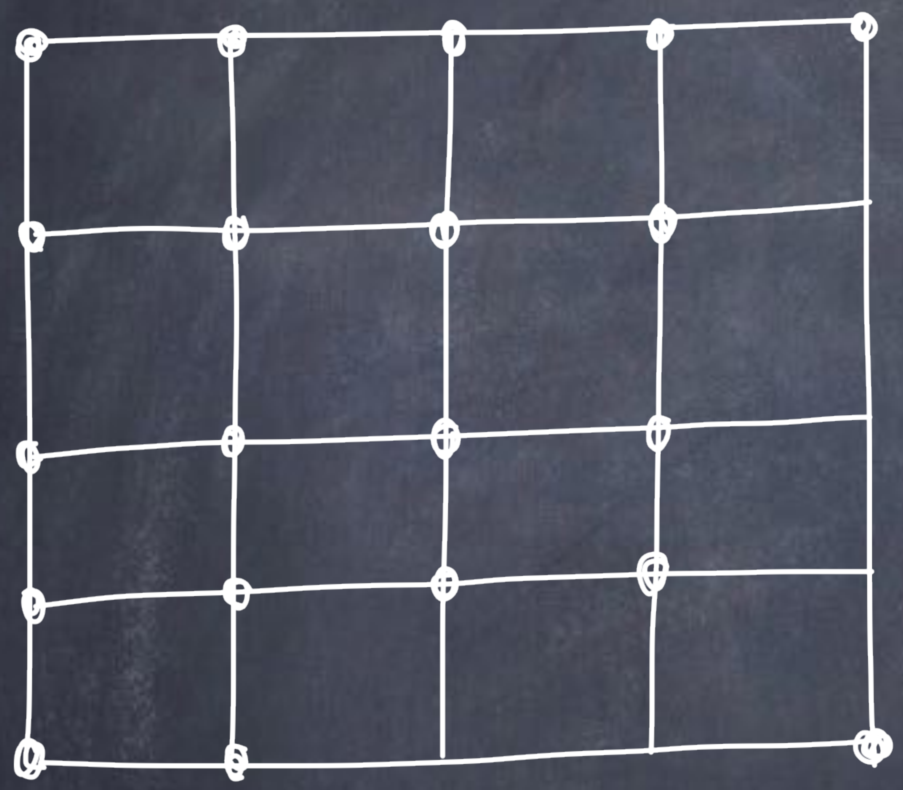
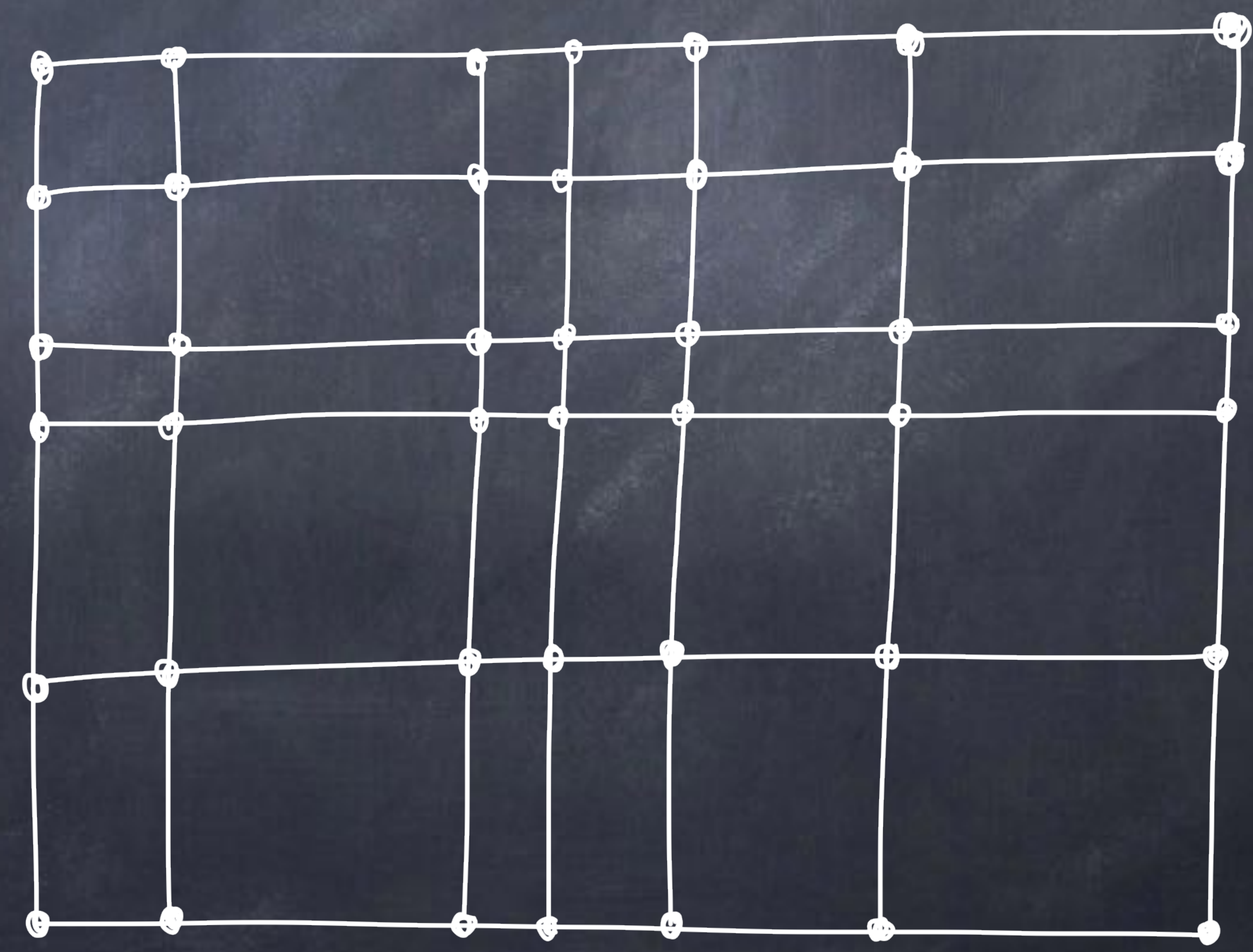


Theorem: (Laman 1970) A graph $G=(V,E)$ is generically rigid if and only if there is a subset F of edges so that $|F| = 2|V|-3$ and $|F'| \leq 2|V(F')|-3$ for all subsets $F' \subseteq F$, where $V(F')$ denotes the set of vertices which are endpoints of F'.

Grid-like frameworks





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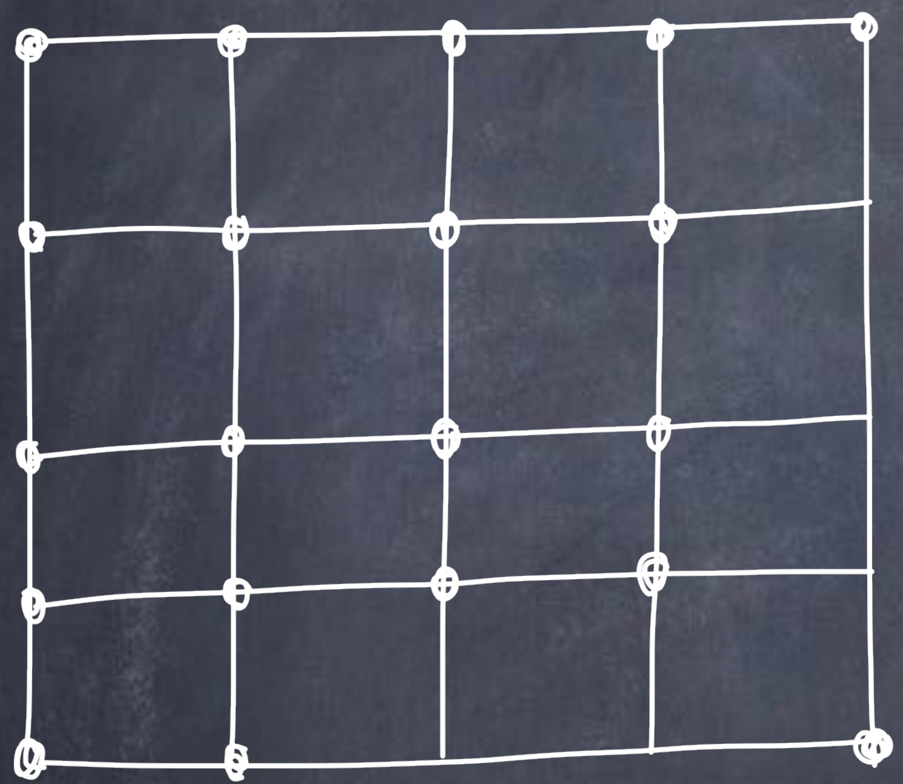
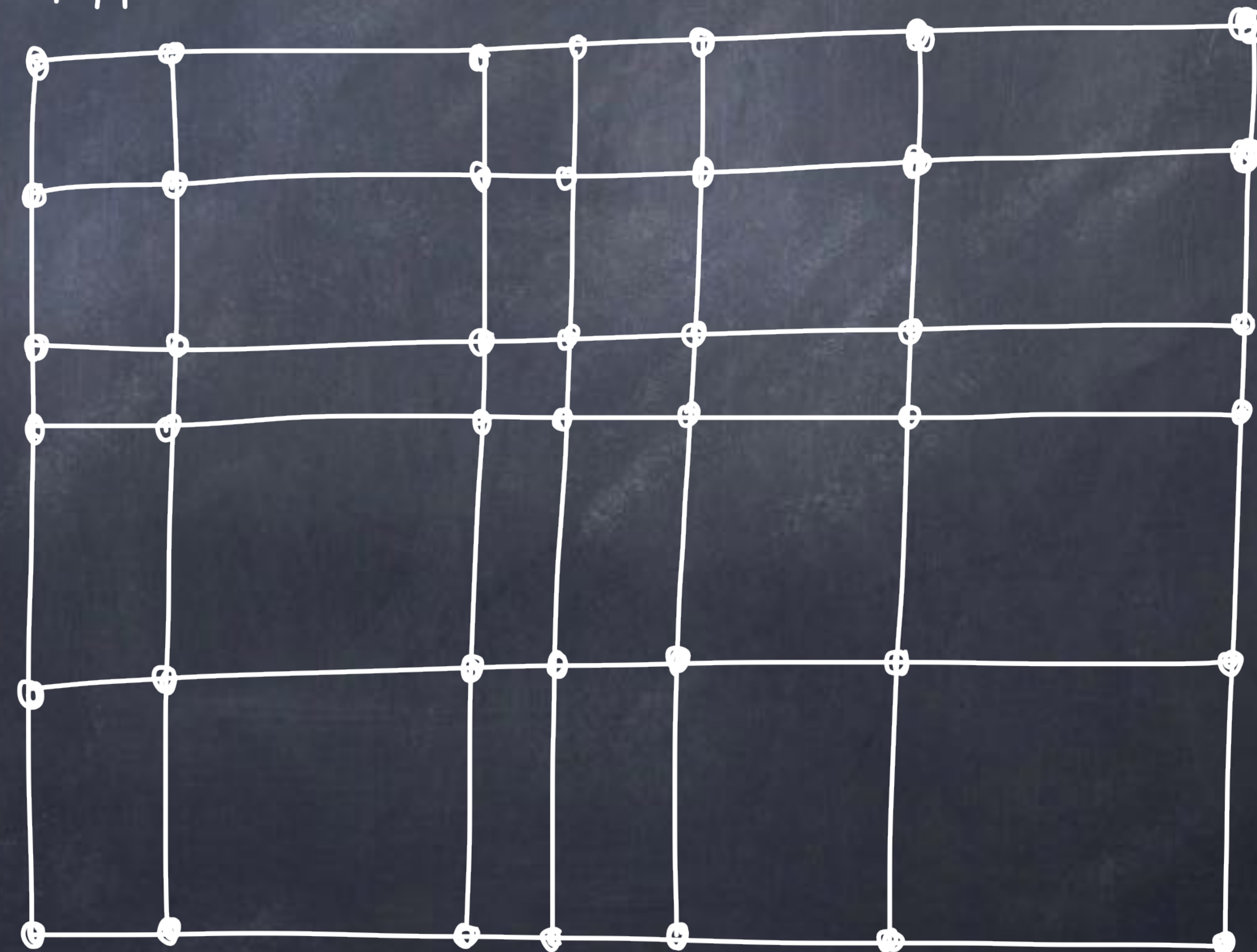
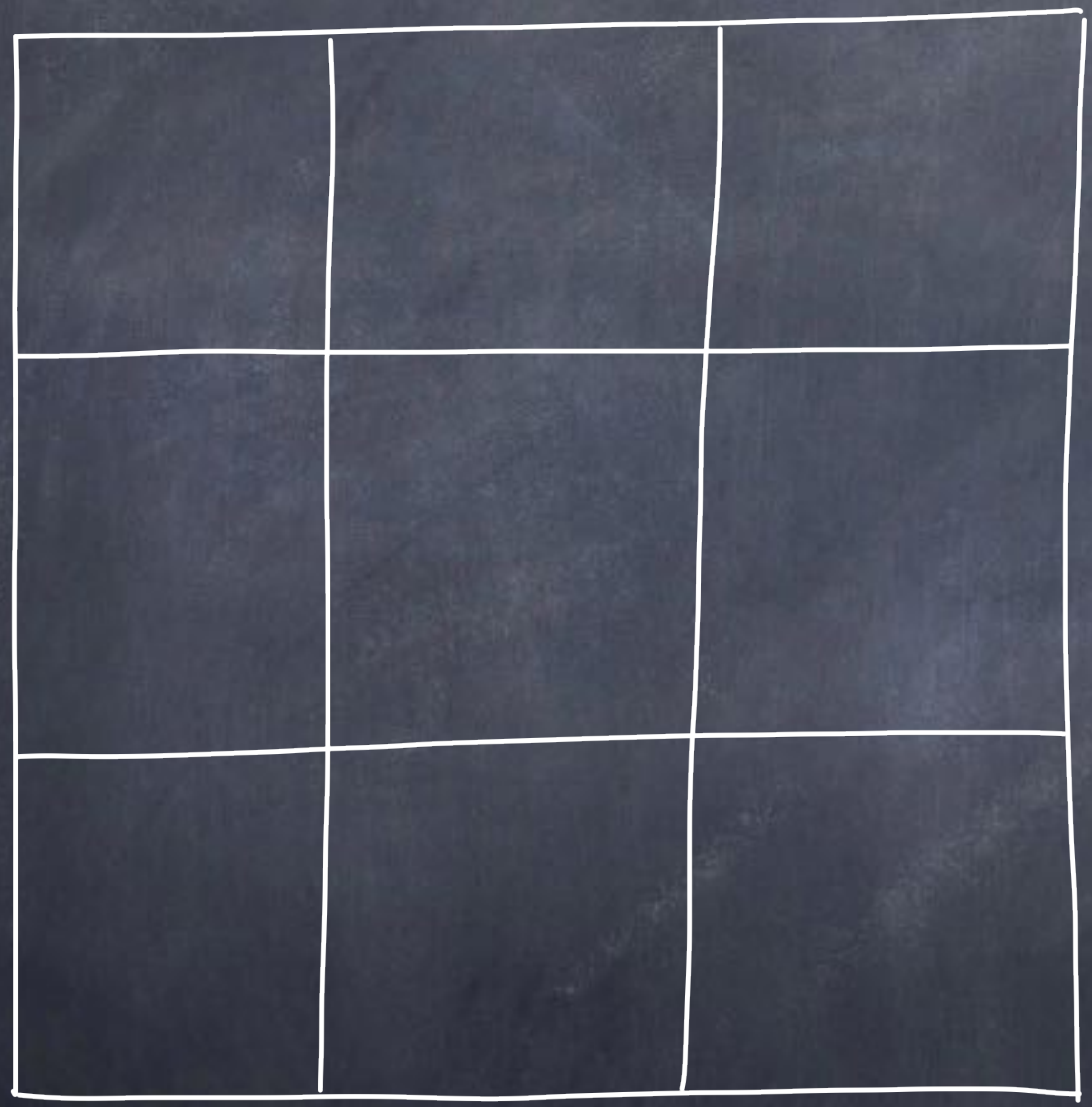


Bolker, Crapo, 79
"Bracing rectangular frameworks"

Radics, Recski, 2002
"Applications of combinatorics to statics-rigidity..."



Rigidity for grids



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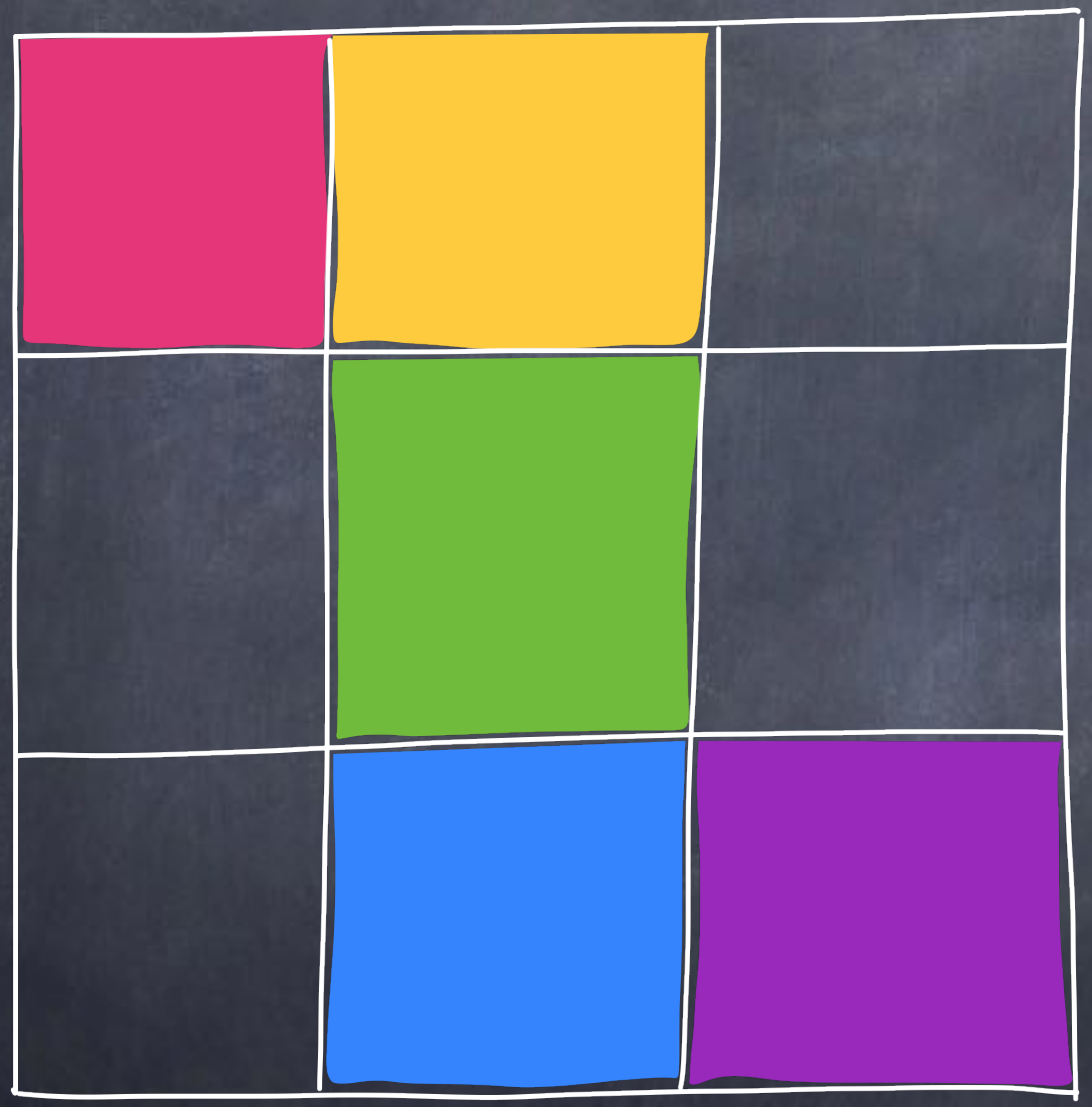

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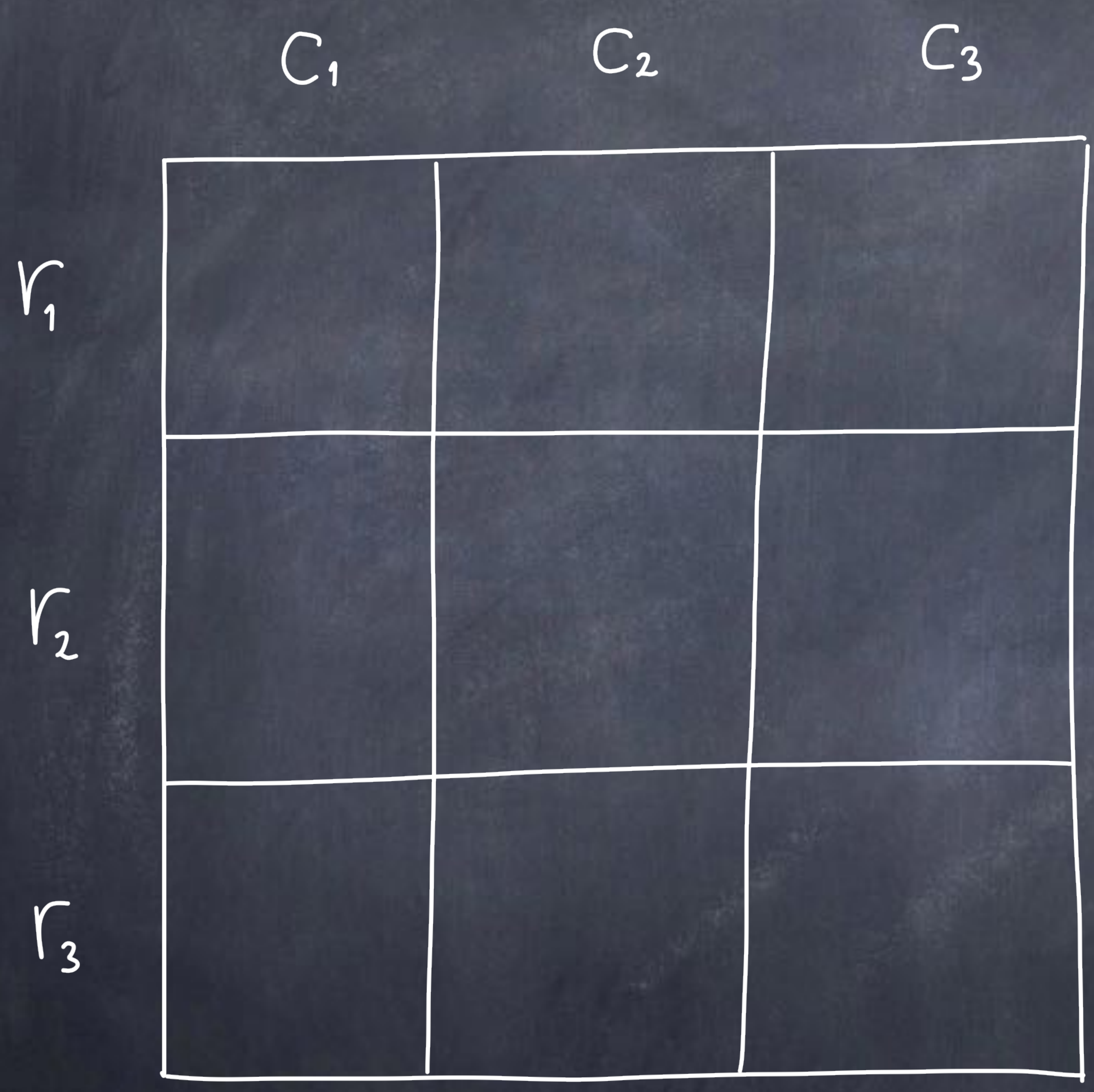

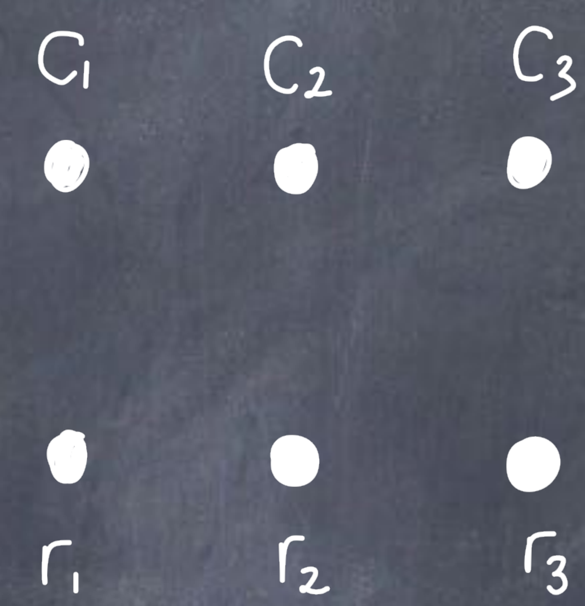

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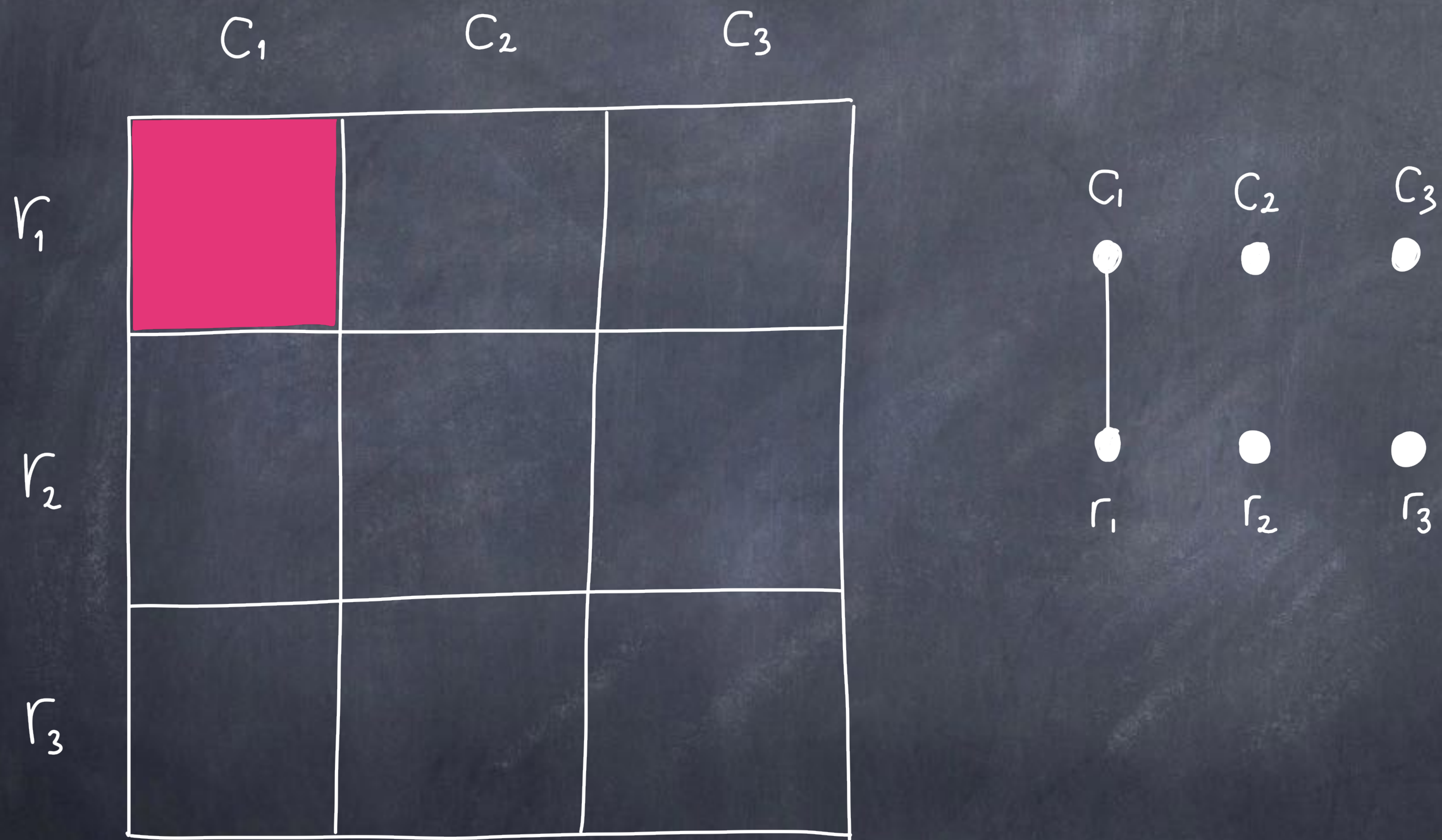

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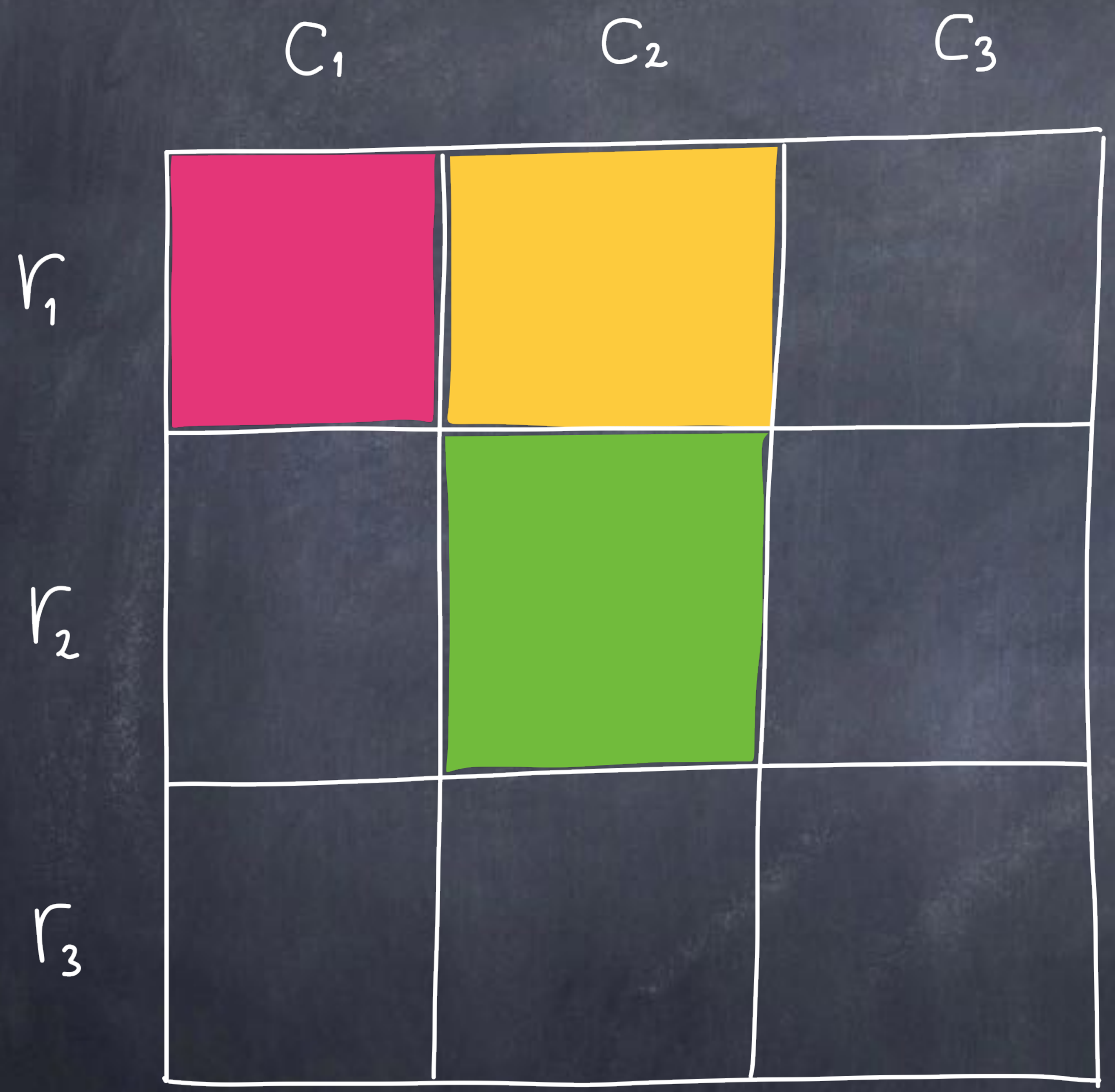

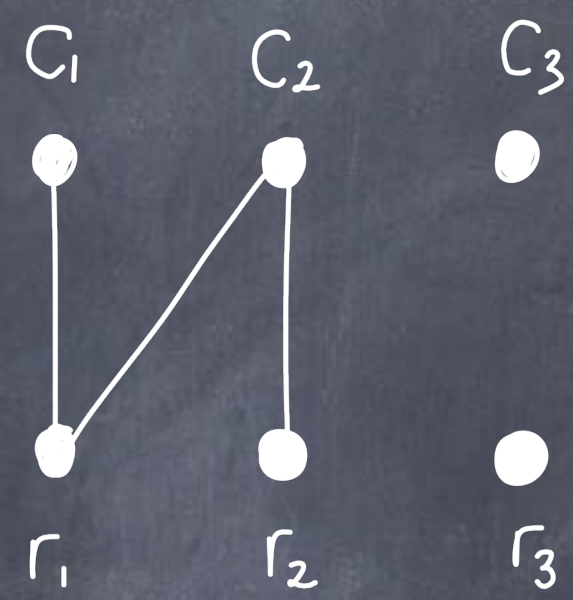

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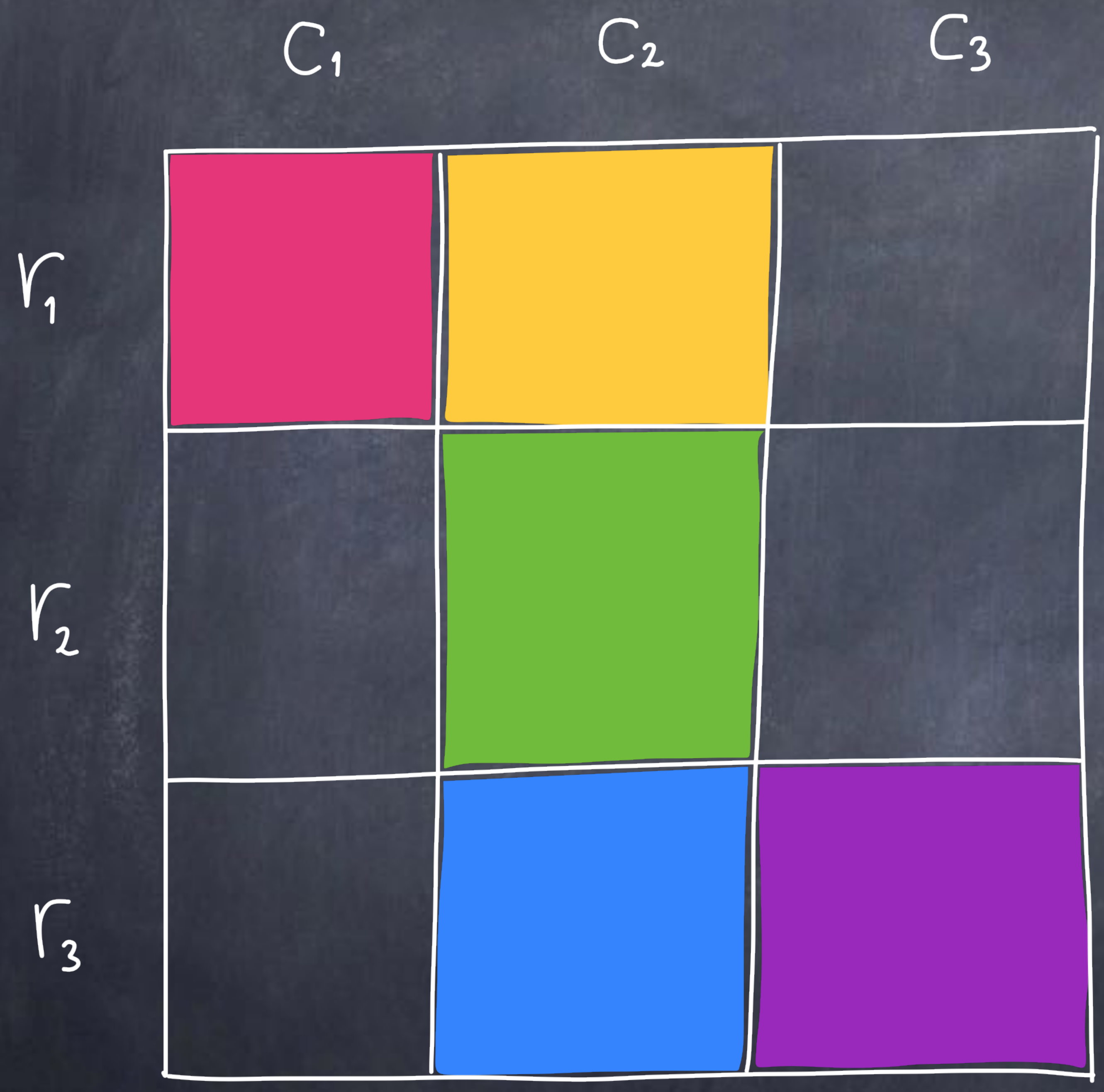
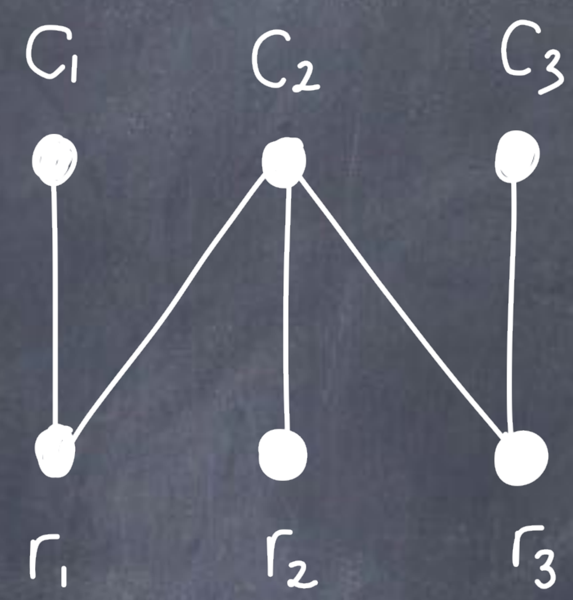
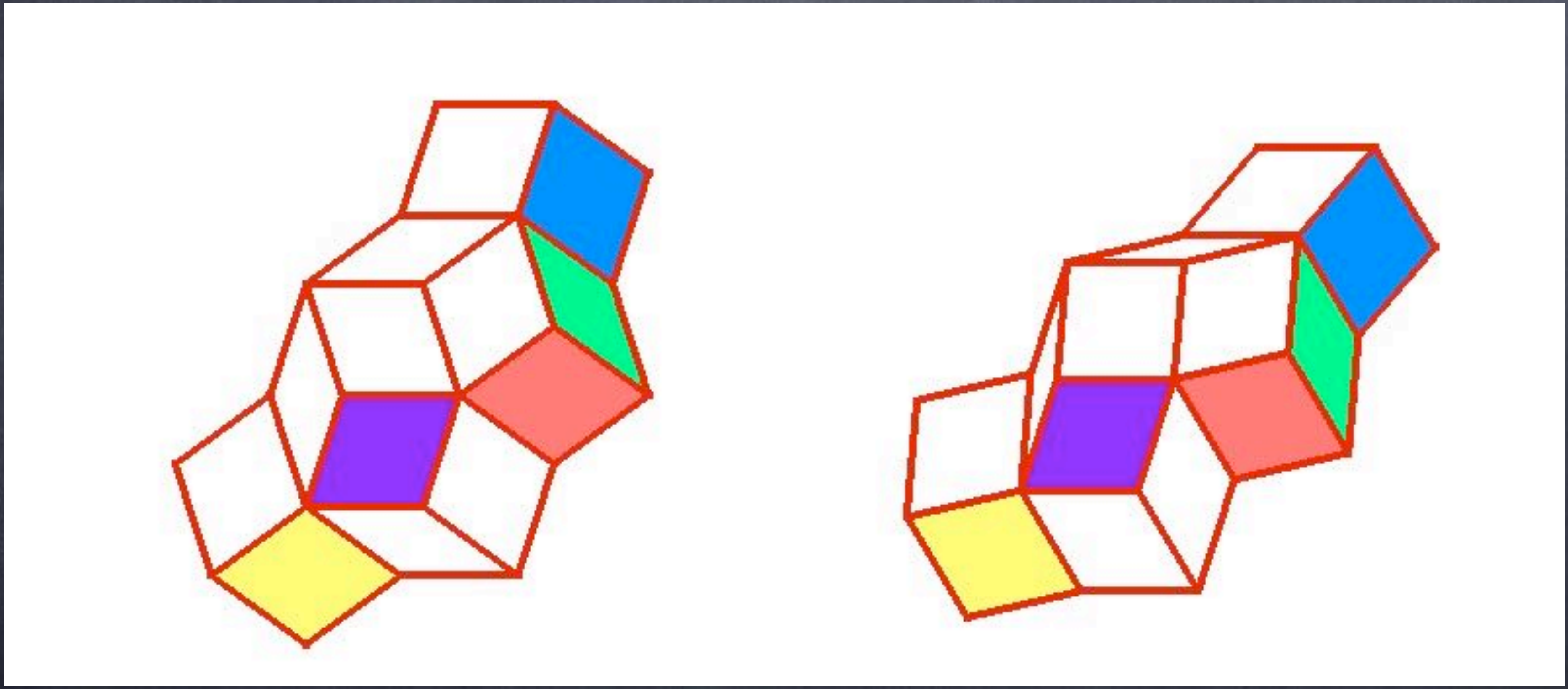




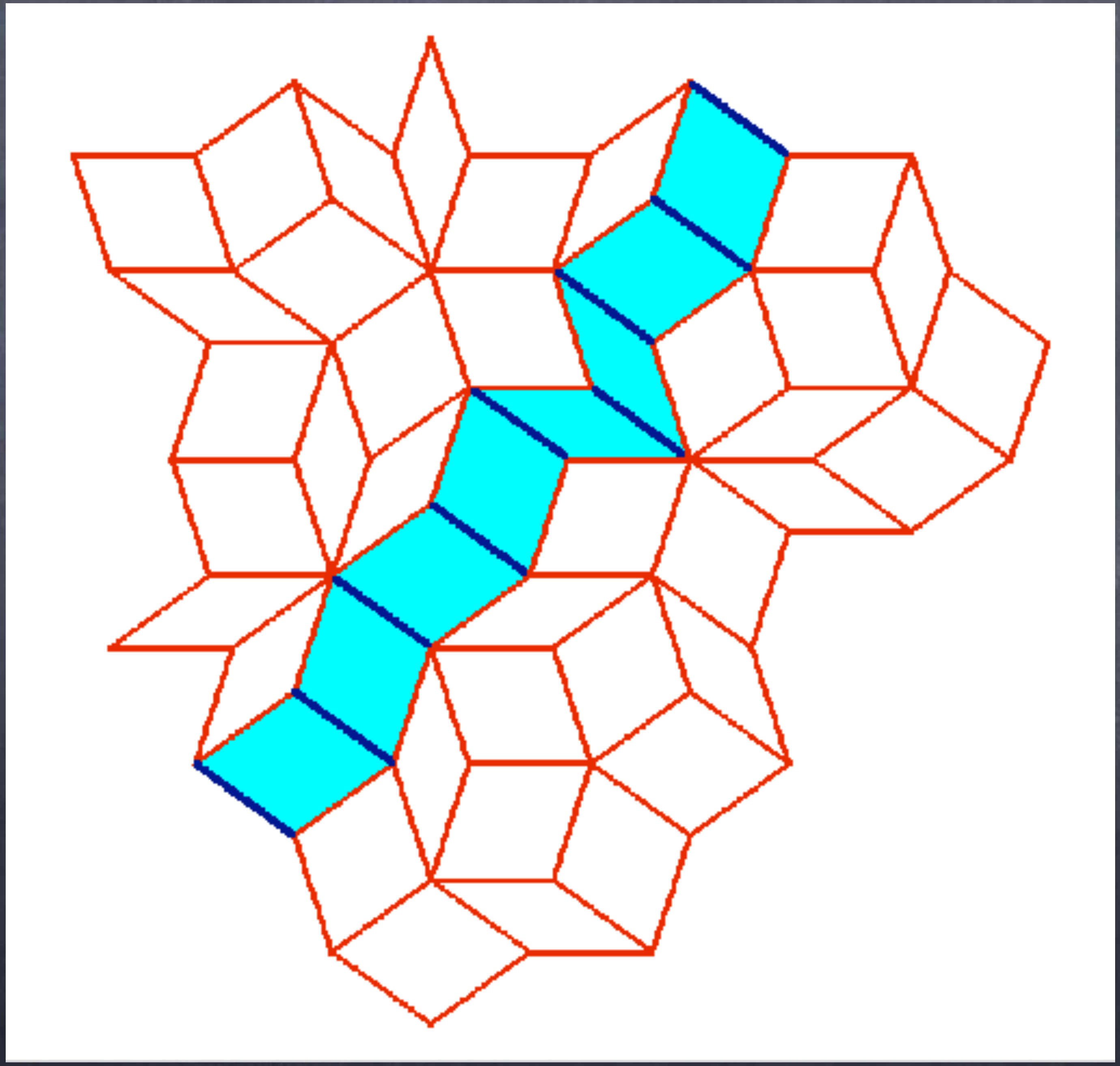
Theorem: (Bolker, Crapo 1979)

A bracing of an $n \times m$ grid is rigid if and only if the corresp. bracing graph is spanning and connected. The bracing is minimum if and only if the bracing graph is a tree.

When is a braced Penrose framework rigid?



Def: In a Penrose framework, a maximal succesion of contiguous rhombi, whose common edges are parallel, is a ribbon.



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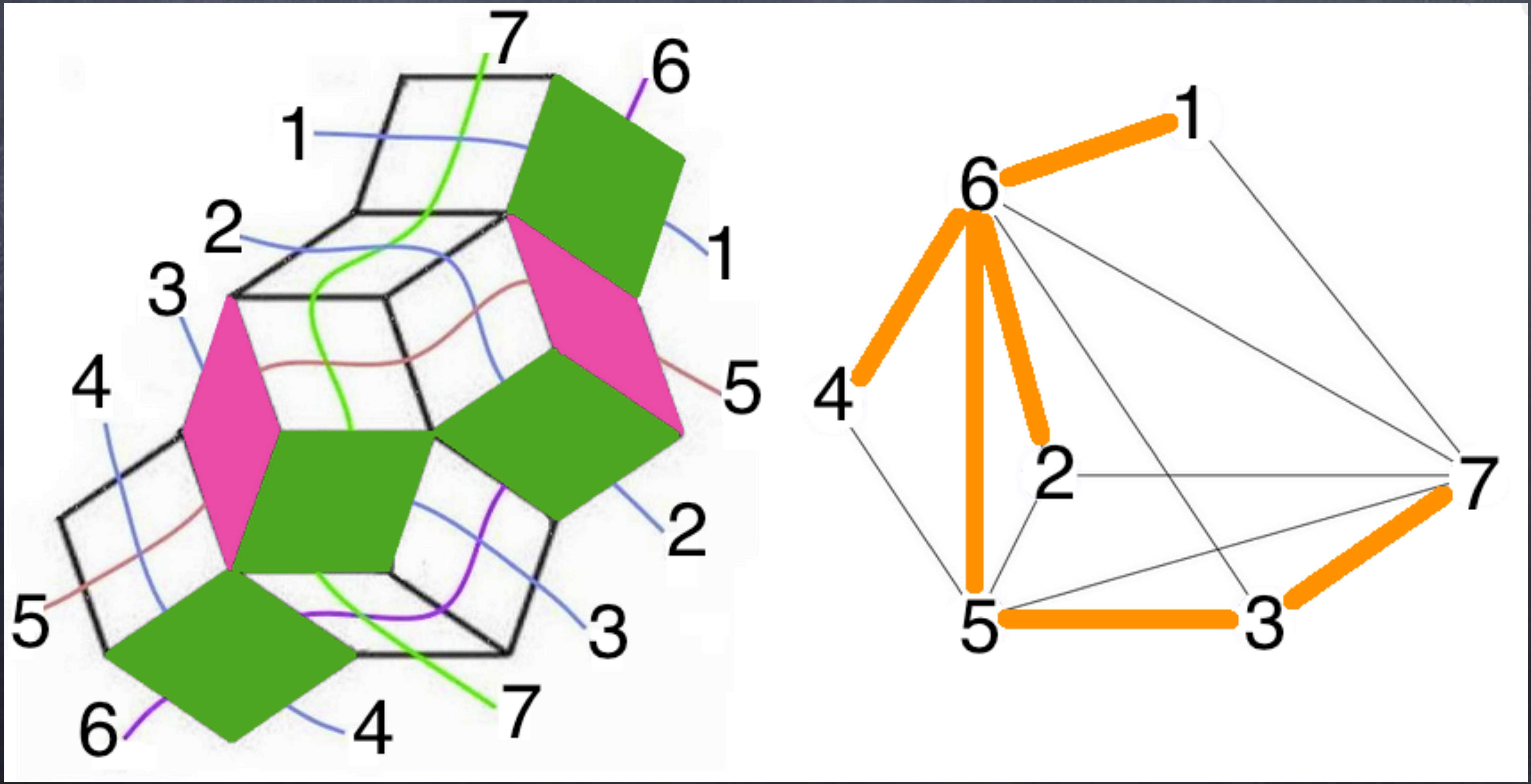

Theorem: (Wester 2006)

A braced Penrose framework is rigid if and only if its bracing subgraph is spanning and connected.

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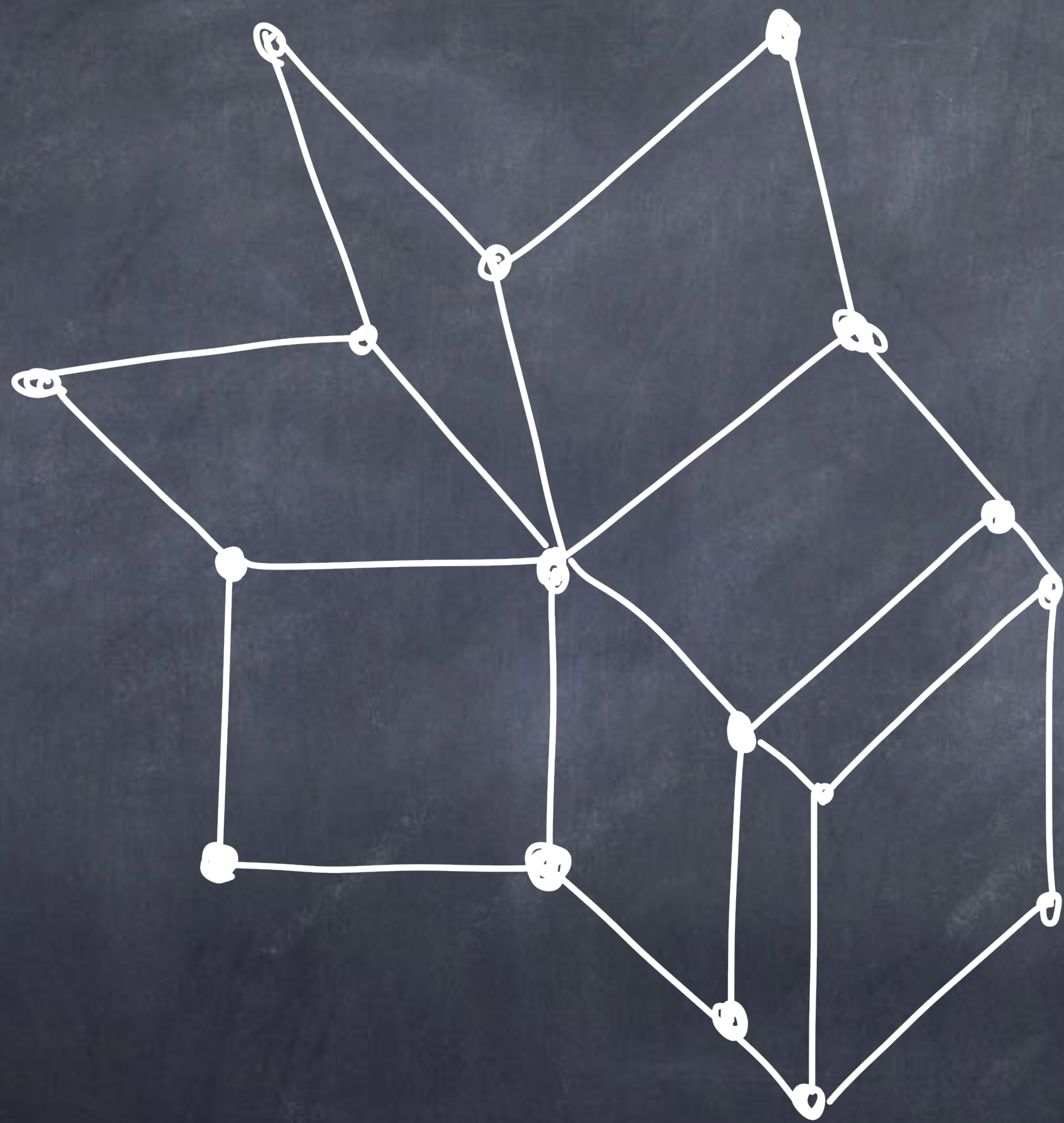
Theorem: (D., Francis 2013)

A simply connected framework made by parallelograms is rigid if and only if its bracing subgraph is spanning and connected.



What happens in 3D??

Grids of cubes → 3D-quasicrystals?

- Recski, 88, "Bracing cubic grids a necesary cond."
- Radics, 99, "Rigidity of t-story buildings".
- Nagy, 94, "Diagonal bracing of special cube gr..."
- Nagy, ~~2006~~ 2019, "Repetitive skeletal structures...."

Grids of cubes ➡ 3D-quasicrystals?

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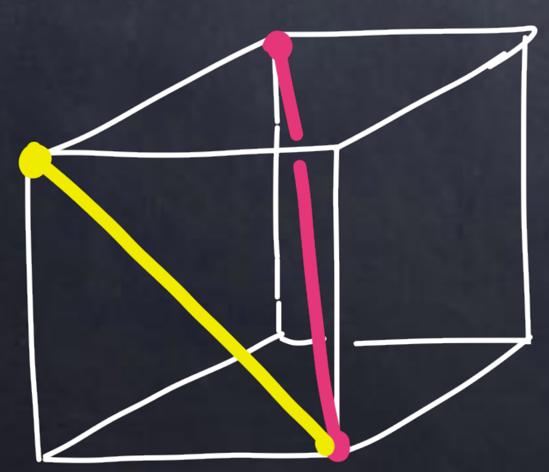
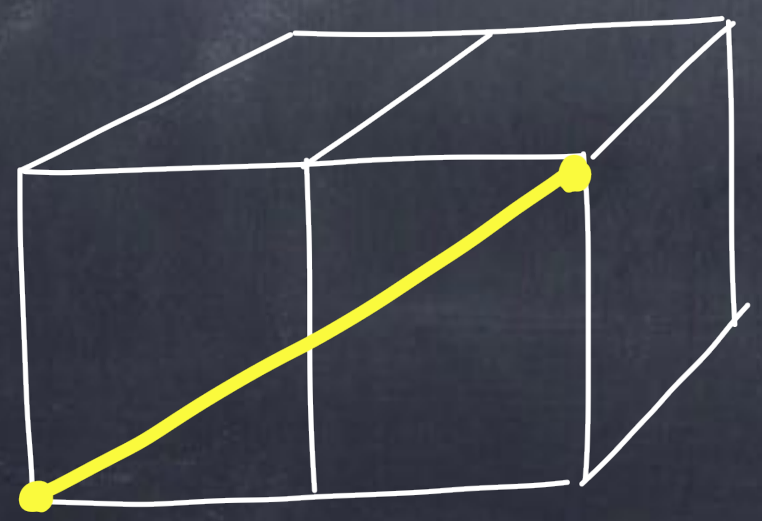
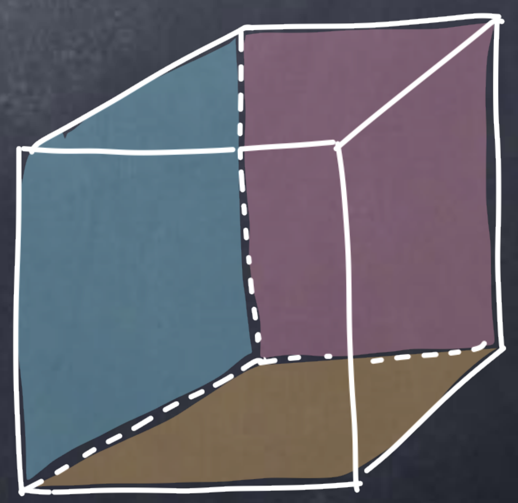
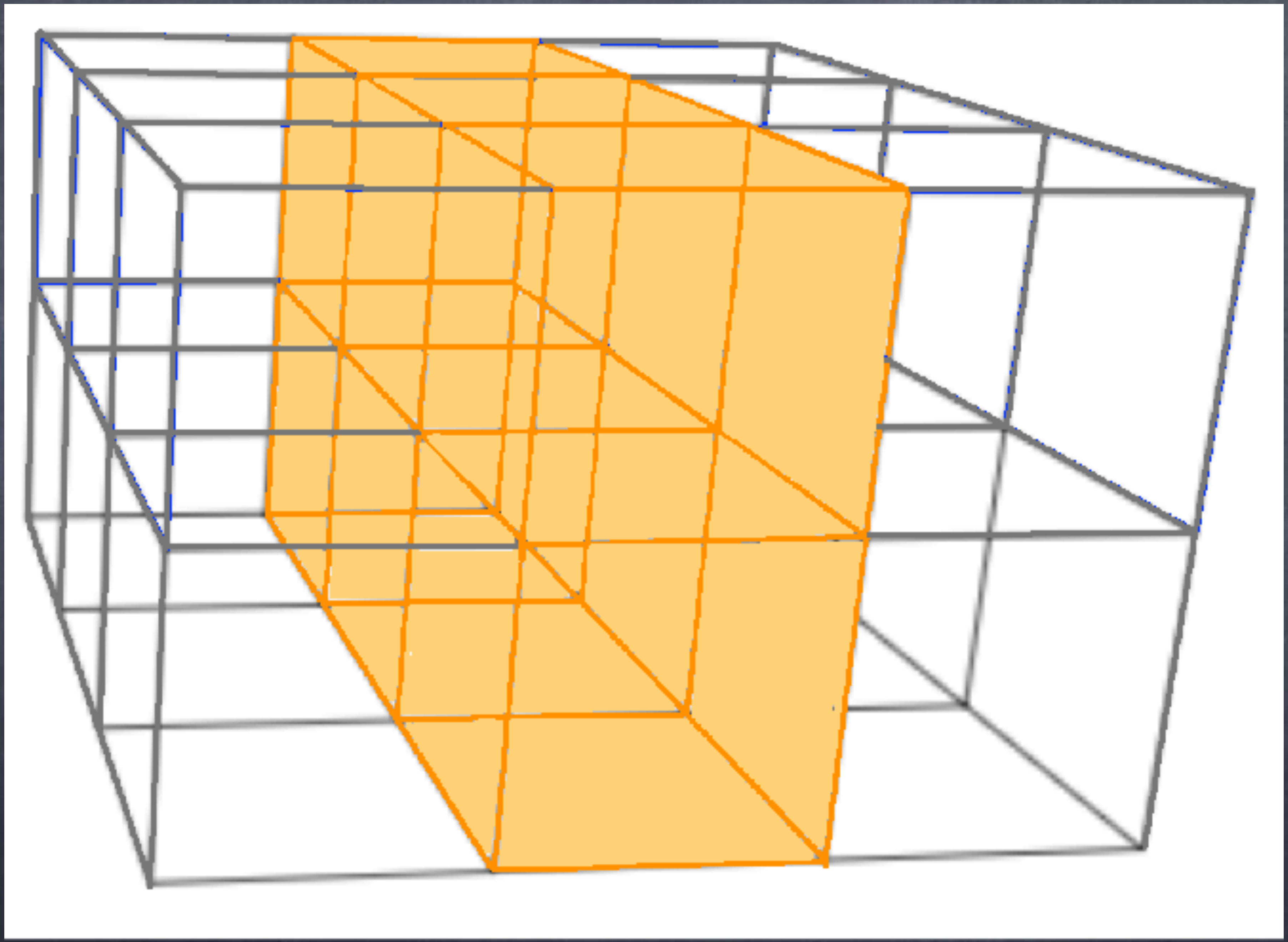


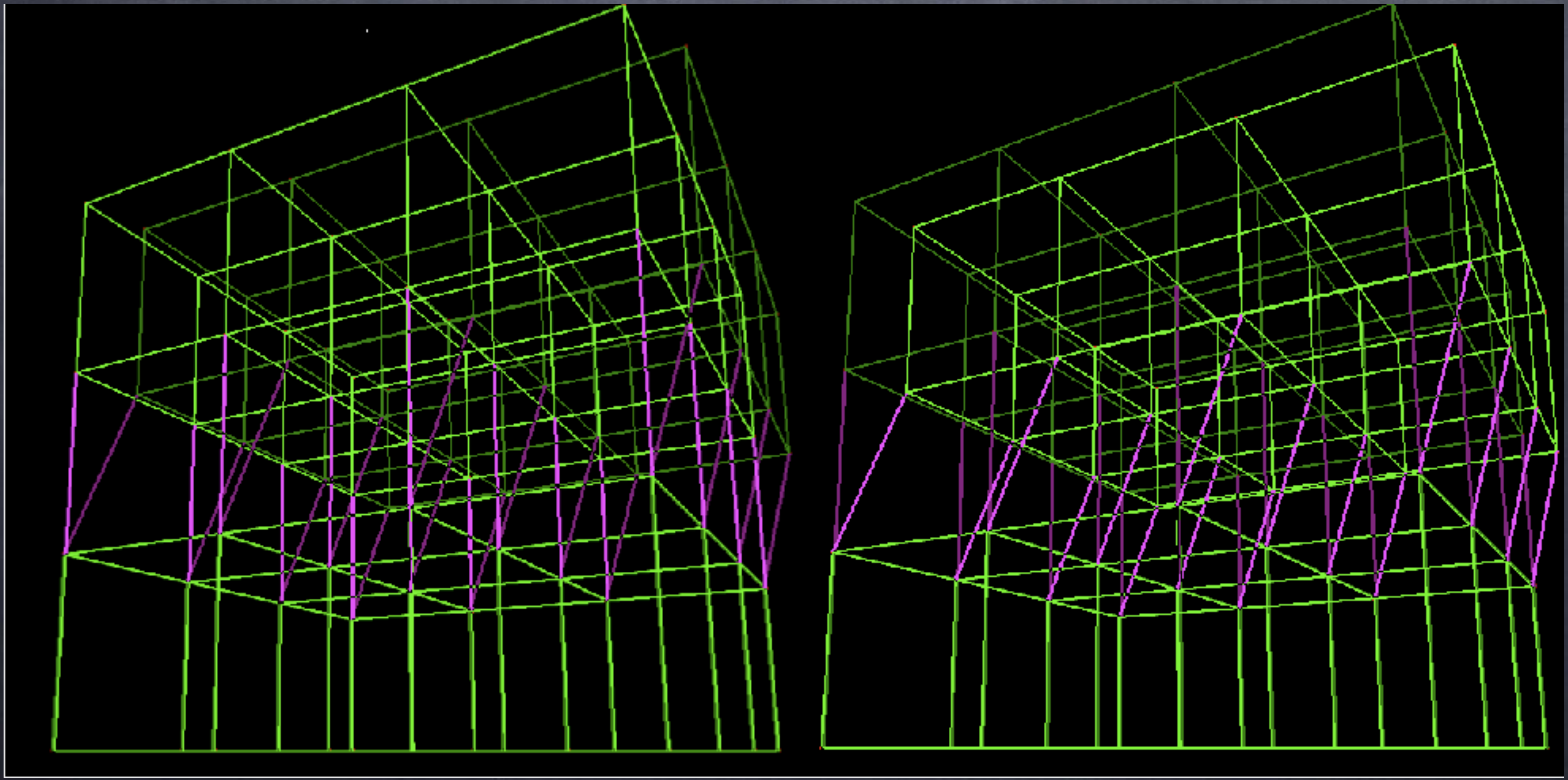




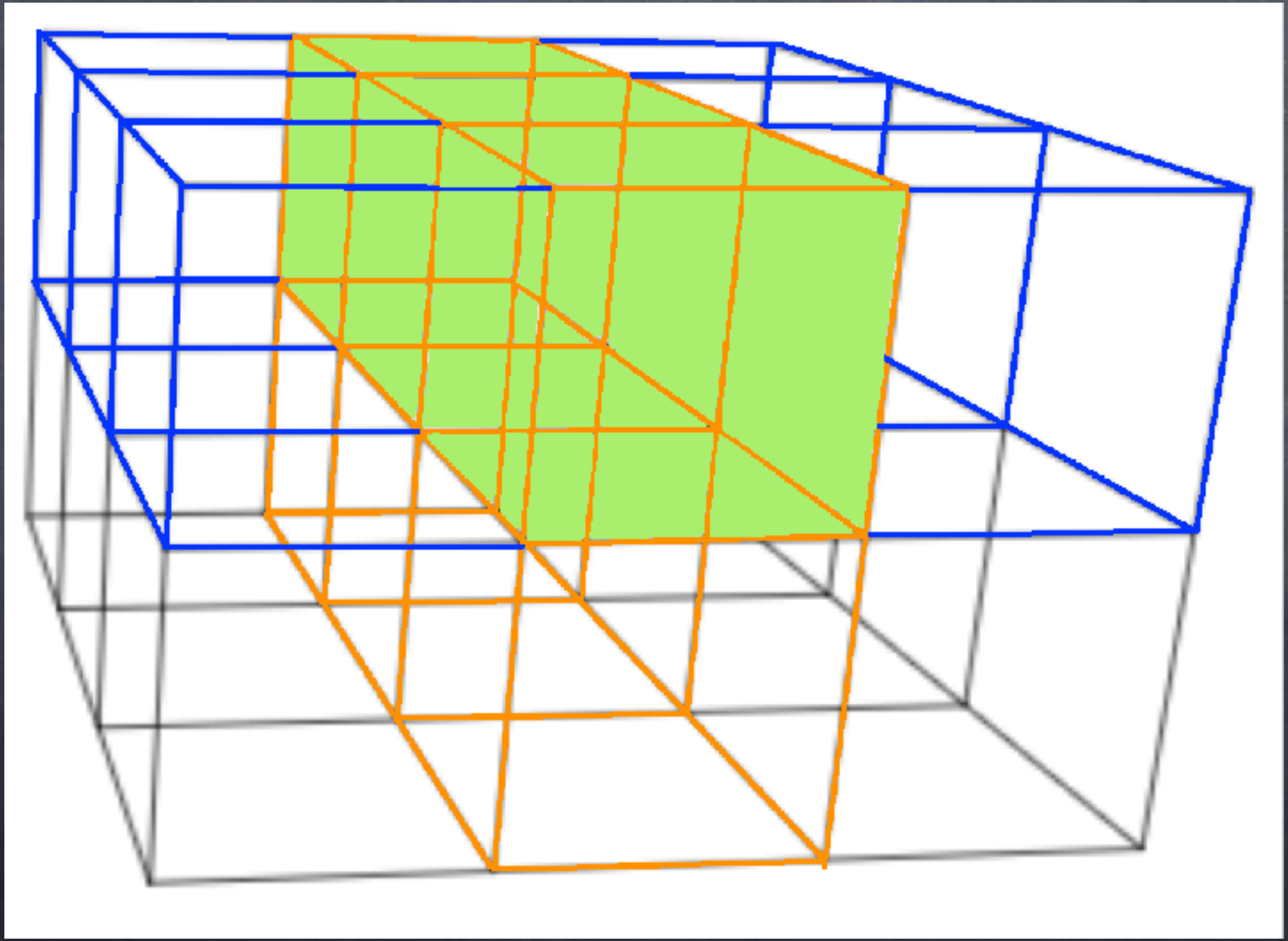
Stratum



Shear along a stratum



Tunnels / Tubes



Question/Conjecture:

A 3D-grid can be made rigid under shears along strata by plating at least one face in each tunnel.

3D Wester Game

http://new.math.uiuc.edu/wester3d/wobble.html

THANK YOU!

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Are there any questions?