Exercise 1 ($*$) Compute the toric blending functions for the unit square

$$P = \mathrm{conv}((0,0),(1,0),(0,1),(1,1)),$$

and prove that they satisfy the sum-to-ne condition. (Hint: use the facet description of this polytope that was given in the lecture, together with the lattice distance functions)

Exercise 2 ($*$) Verify that the toric blending functions for the trapezoid

$$\mathrm{conv}\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}2\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix},\begin{pmatrix}1\\1\end{pmatrix}\right)$$

do not satisfy linear precision. The toric blending functions are given by

$$\tilde{\beta}_{\begin{pmatrix}0\\0\end{pmatrix}} = \frac{(1-y_2)(2-y_1-y_2)^2}{(2-y_2)^2}, \quad \tilde{\beta}_{\begin{pmatrix}1\\0\end{pmatrix}} = \frac{2y_1(1-y_2)(2-y_1-y_2)}{(2-y_2)^2}, \quad \tilde{\beta}_{\begin{pmatrix}2\\0\end{pmatrix}} = \frac{y_1^2(1-y_2)}{(2-y_2)^2},$$

$$\tilde{\beta}_{\begin{pmatrix}0\\1\end{pmatrix}} = \frac{y_2(2-y_1-y_2)}{2-y_2}, \quad \tilde{\beta}_{\begin{pmatrix}1\\1\end{pmatrix}} = \frac{y_1y_2}{2-y_2}.$$

Exercise 3 ($*$) Give a rational parametrization for the ML degree one locus for the second Veronese surface $\nu_2(\mathbb{P}^2)$. Start by finding a polynomial $F(x,y,z)$ whose monomials appear as coordinates of the Veronese map ν_2.

Exercise 4 ($*$) Prove that F defines a toric polar Cremona transformation if and only if F^a defines a toric polar Cremona transformation for all $a \in \mathbb{N}$.

Exercise 5 ($**$) Verify that the polynomial $F = x_0(x_0x_2 + x_1^2)$ is *homaloidal*, i.e. the map $\mathbb{P}^2 \dashrightarrow \mathbb{P}^2, \quad (t_0 : t_1 : t_2) \mapsto \left(\frac{\partial F}{\partial x_0}(t) : \frac{\partial F}{\partial x_1}(t) : \frac{\partial F}{\partial x_2}(t)\right)$ is birational.